Time Domain Repetitive Analysis of Analog Circuits with Transmission Lines

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Abstract - **This paper presents the repetitive analysis of circuits containing elements described by distributed parameters, such as transmission lines. The time domain repetitive analysis is performed with the utilisation of a polynomial number method and symbolic formulas, and is fast enough to show animated graphs of circuit response according to change of some parameters of the circuit.**

1. INTRODUCTION

The repetitive time domain analysis it is a calculation of the response function of the circuit, for example step response, while we change some parameters of the circuit many times. A way to achieve a high speed of this type of calculation is to determine the response function as the numerical and symbolic formula, where the numerical coefficients depend on the circuit topology and the value of the fixed (non-symbolic) parameters while only variables are the symbolic parameters [1]. To describe the response function as the numerical and symbolic expression the operational calculus can be used ([2], [3], [4]) and a polynomial number method ([5], [6]) can be utilised (table 1).

Type of signal	*Equivalent number*	*Computer implementation*
DC	real number	real number (N bits)
AC	complex number	2 real numbers
transient	polynomial number	M real numbers and integer number

Table 1. Signals and numbers

It is important that the polynomial number method can be used not only for the lumped circuits. Also the elements modelled using the distributed parameters can be taken into account.

2. TRANSMISSION LINE MODEL

Starting from differential equations of voltages and currents (fig. 1)

$$\begin{aligned} \frac{d}{dx}u(x) &= -Z'\,i(x) \\ \frac{d}{dx}i(x) &= -Y'\,u(x) \end{aligned} \qquad (1)$$

where $u(x)$, $i(x)$ are operators of time domain functions and, according to denotation entered by Mikusinski in [7] and [8], they can be written as:

$$u(x) = \{u(x, t)\},\ i(x) = \{i(x, t)\}\ . \qquad (2)$$

Z', Y' are transmission line parameters per unit length and are functions of Heaviside's operator

$$p = \frac{1}{\int_0^t}\ . \qquad (3)$$

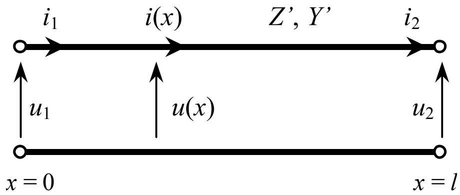

Fig. 1. Transmission line voltages and currents.

In a simple case $Z' = R' + pL'$, $Y' = G' + pC'$, but there are more complicated cases when Z', Y' are described by Bessel functions [10] and then series of $\sqrt{p}$, as in [9] should be considered.

Connections between the near-end and far-end voltages and currents can be expressed by equations:

$$\begin{aligned} i_1 &= Y_c\,u_1 - Y_c e^{-\gamma l}\,u_2 + e^{-\gamma l}\,i_2 \\ i_2 &= -Y_c\,u_2 + Y_c e^{-\gamma l}\,u_1 + e^{-\gamma l}\,i_1 \end{aligned} \qquad (4)$$

where $Y_c = \sqrt{Y'/Z'}$, $\gamma = \sqrt{Y'Z'}$. The equivalent circuit diagram corresponding to these equations is shown in fig. 2.

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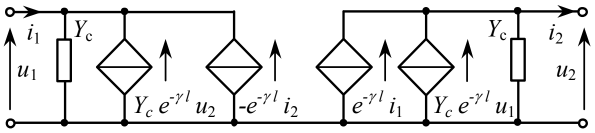

Fig. 2. Transmission line model.

The controlled current sources in the transmission line model contain such coefficients as $e^{-\gamma l}$ or $Y_c e^{-\gamma l}$ which are complicated functions of Heaviside's operator, due to square roots and exponential functions. All these functions can be calculated using the polynomial number algorithms [5], because basic operations +, -, /, *, as well as functions such as: rising to any rational power, exp(), ln(), sin(), cos() can be calculated in polynomial number domain.

3. Z-TRANSFORM AND EXPONENTIAL FUNCTION

Fast calculation of numerical-symbolic formulas forces to use Z-transform in a polynomial number calculator ([11], [5], [1]). The Heaviside's operator p (3) can be approximated by a polynomial number $\underline{p}$, for example in trapez rule case:

$$\underline{p} = \frac{2}{h} \frac{(\tilde{}1\tilde{}, -1\tilde{})}{(\tilde{}1\tilde{}, 1\tilde{})} \tag{5}$$

where h is the sampling period. As it was shown in [5], the exponential function with an argument, which is a complicated function of $\underline{p}$ can be calculated more accurate when "pure" delay operator is separated from the argument. For example, calculation of function

$$F(p) = \frac{1}{p^2 + p + 4} \exp(-4\sqrt{p^2 + 1}) \tag{6}$$

can be more accurate when pure delay operator exp(- $T_{sh}\, p$) is separated. This delay operator can be accurately expressed in Z transform, because it corresponds to the shift of the sequence, so series of samples satisfies formula:

$$\begin{aligned} \underline{f} &= F(\underline{p}) \cdot \underline{p} \cdot (\tilde{}0.5\tilde{}, 1\tilde{}1\tilde{}\ldots\tilde{}) = \\ &= (\tilde{}1\tilde{}0\tilde{})^{-N} \frac{1}{\underline{p}^2 + \underline{p} + 4} \exp(-(4\sqrt{\underline{p}^2 + 1} - Nh\underline{p})) \cdot \\ &\cdot \underline{p} \cdot (\tilde{}0.5\tilde{}, 1\tilde{}1\tilde{}\ldots\tilde{}) \end{aligned} \tag{7}$$

where $N = \left\lfloor \frac{T_{sh}}{h} \right\rfloor$, $\lfloor\ \rfloor$ - integer part of the number.

As it is illustrated in fig. 3, too small delay T_{sh} causes too small accuracy (it can be noticed, that sample period h=0.3 is relatively high, nevertheless samples are close to $f(t)$).

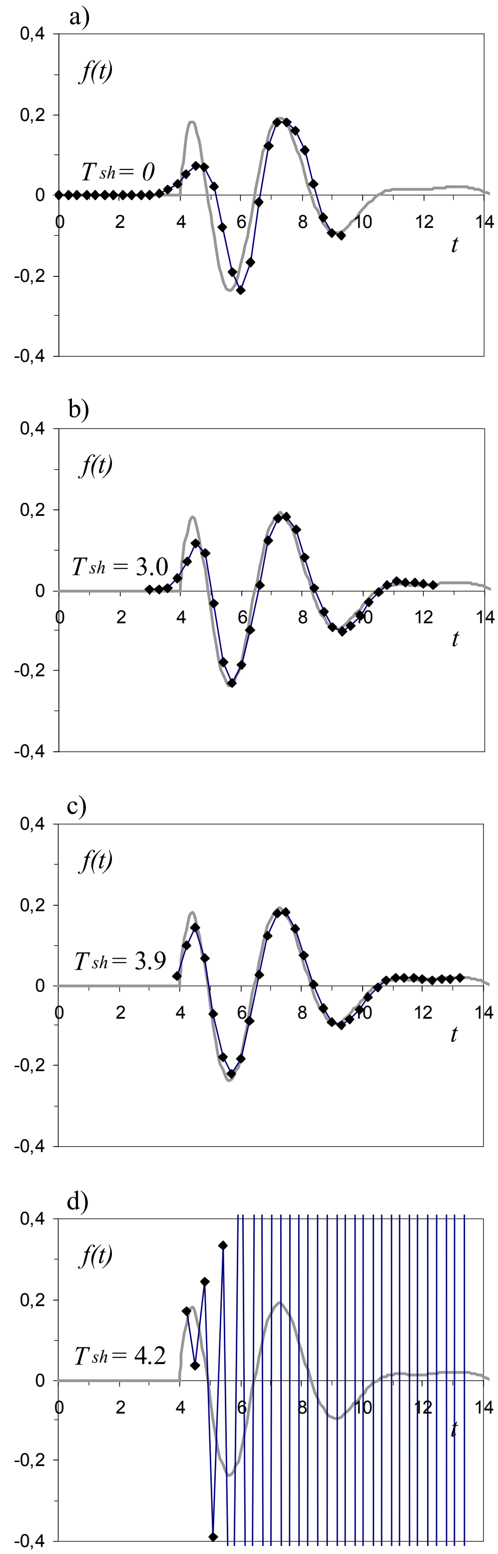

Fig. 3. Inverse Laplace transform ( ) and Z transform samples (, sampling period h=0.3) with different pure delay operator T_{sh} .

Too high delay causes strange effect, as in fig. 3d, so it is very important to set a proper value of T_{sh} time. But in a general case delay values are hidden in the arguments of the exponential functions, so an additional algorithm should be used to determine delay value. The idea is based on multiply calculations of exp() function – precalculation should be used to determine optimal delay time T_{sh}. Iterations should start from T_{sh}=0 (fig. 3a), and T_{sh} for next iteration can be determined, as the time when the samples differ from zero (fig. 3b). Next iterations should continue with higher T_{sh} until samples becomes unrealistic (very high absolute values – see fig. 3d).

The strange values of the Z-transform PN-digits in fig. 3d are not an illustration of the numerical instability, but are correct values correspond to eq. (7). This polynomial number acts as operator, with "negative delay" that means, when it will be multiply with another polynomial number containing "positive delay" operator it will give real samples in the result (see remarks of Mikusinski in [8] on page 135). The series of the samples correspond to operator do not approximate any function $y(t)$ – when sampling period is changed then new samples are quite different from previous series. An example for a dependence of the Heaviside's operator samples and sampling period h is shown in the fig. 4. Similar property concerns the samples of operator with "negative delay" in fig. 3d.

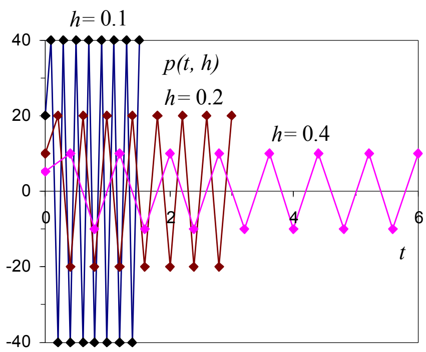

Fig. 4. Different Heaviside's operator samples (5) with different sampling period values.

4. NUMERICAL EXAMPLE

An example of utilizing transmission line model shows the transient repetitive analysis with symbolic elements in RAN5 program (Repetitive Analysis of Networks). Output voltage in a circuit in fig. 5 can be derived from formula (4) (all symbols in the formula (8) except l and k are functions of p):

$$u_2 = \frac{\frac{1}{2}(1-\rho_1)(1+\rho_2)\exp(-\gamma l)}{1-\rho_1\rho_2\exp(-2\gamma l)}u_0 \qquad (8)$$

where $\rho_{1,2} = \dfrac{Y_c - Y_{1,2}}{Y_c + Y_{1,2}}$.

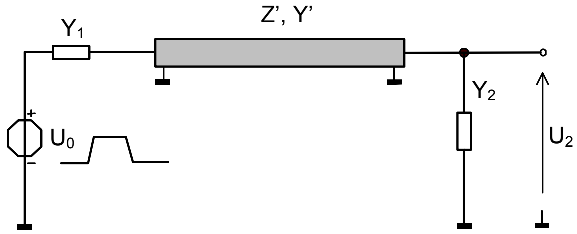

Fig. 5. Linear circuit with transmission line.

The polynomial number $\underline{u}_2$ with digits equal approximately to samples of $u_2(t)$ is calculated in program RAN5 (fig. 6) using $u_2(\underline{p})$ from (8) with $\underline{p}$ as in (5):

$$\begin{aligned}
&\underline{u}_2 = u_2(\underline{p})\cdot\underline{p}\cdot(\tilde{}0.5\tilde{},1\tilde{}\,1\tilde{}\ldots\tilde{}) = \\
&= \frac{\frac{1}{2}\cdot(1-\rho_1(\underline{p}))\cdot(1+\rho_2(\underline{p}))\cdot\exp(-\gamma(\underline{p})\cdot l)}{1-\rho_1(\underline{p})\cdot\rho_2(\underline{p})\cdot\exp(-2\cdot\gamma(\underline{p})\cdot l)}\cdot \\
&\cdot u_0(\underline{p})\cdot\underline{p}\cdot(\tilde{}0.5\tilde{},1\tilde{}\,1\tilde{}\ldots\tilde{}) = \\
&= \frac{\frac{1}{2}\cdot(1-\underline{\rho}_1)\cdot(1+\underline{\rho}_2)\cdot\exp(-\underline{\gamma}\cdot l)}{1-\underline{\rho}_1\cdot\underline{\rho}_2\cdot\exp(-2\underline{\gamma}\cdot l)}\underline{u}_0
\end{aligned} \qquad (9)$$

where $\underline{u}_0 = u_0(\underline{p})\cdot\underline{p}\cdot(\tilde{}0.5\tilde{},1\tilde{}\,1\tilde{}\ldots\tilde{})$ is the polynomial number with digits equal to $u_0(t)$ samples. Even for all parameters of the circuit and all parameters of transmission line treated as symbols this calculation is fast enough to show animated graphs for $u_2(t)$. 128 samples are calculated in time 1.8ms using computer PC with Pentium 1.7GHz.

5. CONCLUSIONS

The polynomial number method refers to the computer implementation of some operational calculi and is useful in the transient repetitive analysis, where the pre-generated numerical-symbolic formulas should be used. A range of utilisation of this method incorporates systems with elements described by the distributed parameters.

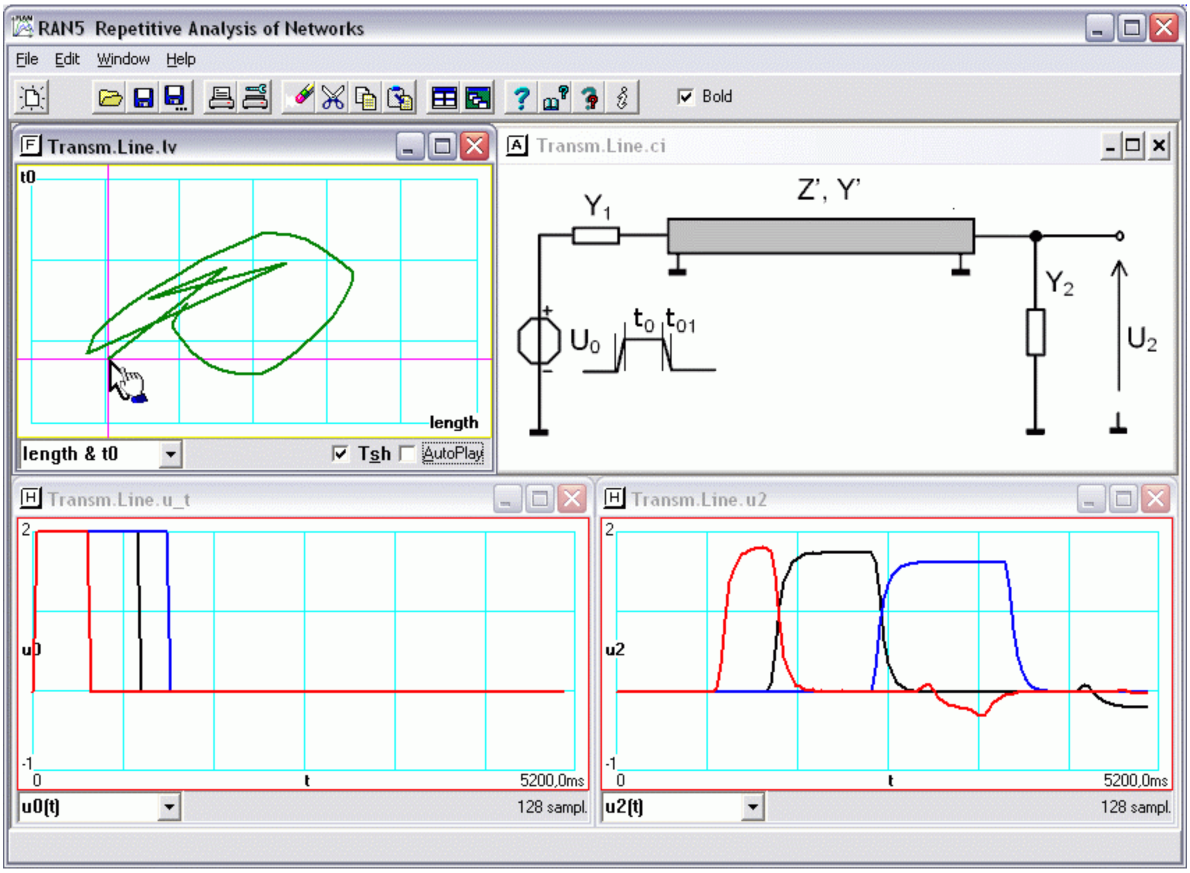

Fig. 6. RAN5 – Repetitive Analysis of Networks with transmission lines

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Note:
Full text versions of some mentioned above papers are available at http://www.pei.prz.rzeszow.pl/~kubaszek